SADLER MATHEMATICS METHODS UNIT 3

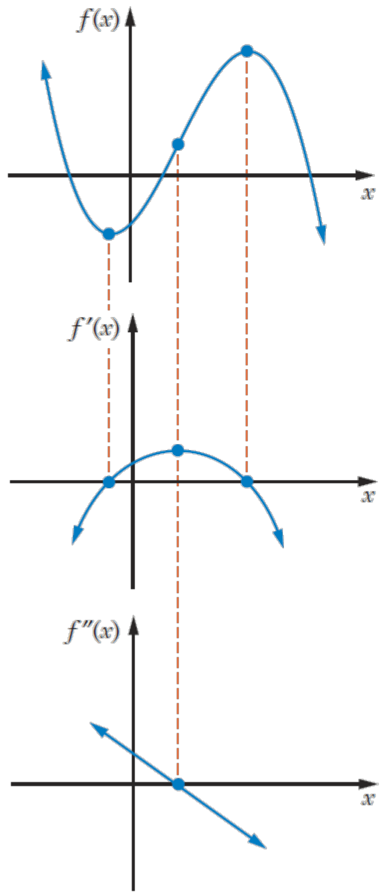
WORKED SOLUTIONS

Chapter 2 Applications of differentiation

Exercise 2A

Question 1

a



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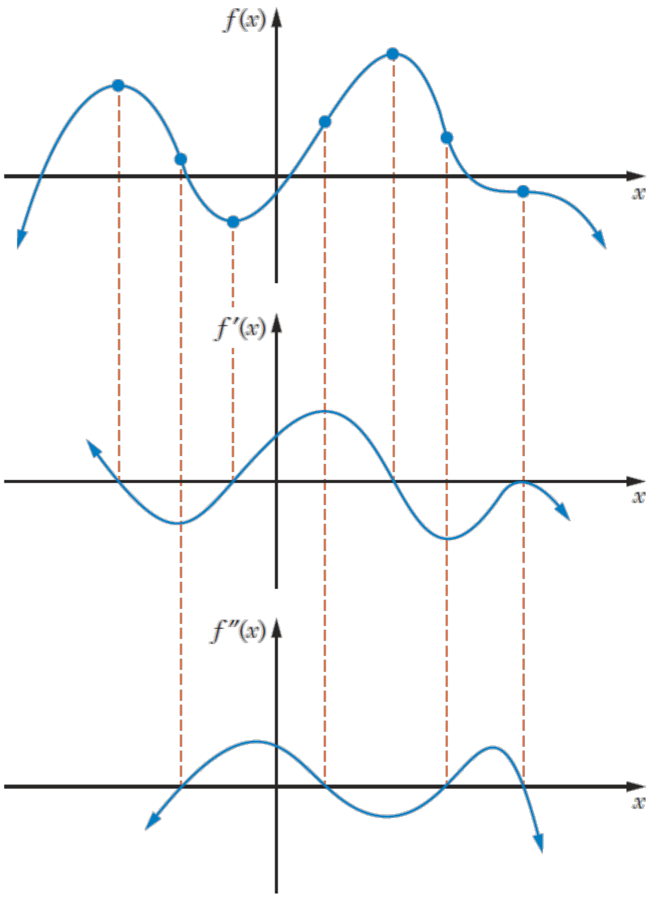


Question 2

$y = x^2 - 12x + 40$

$\frac{dy}{dx} = 2x - 12$

Stationary points occur when $x = 0$.

$2x - 12 = 0$

$x = 6$

When $x = 6$,

$y = 6^2 - 12(6) + 40$

$= 4$

$\therefore$ A stationary point exists at $(6,\ 4)$.

$\frac{d^2y}{dx^2} = 2$

Given $\frac{d^2y}{dx^2} > 0$, $(6,\ 4)$ is a minimum point.

Question 3

$y = 5 + 8x - x^2$

$\frac{dy}{dx} = 8 - 2x = 0$

$x = 4$

When $x = 4$,

$y = 5 + 8(4) - 4^2$

$= 21$

There is a stationary point at $(4,\ 21)$.

$\frac{d^2y}{dx^2} = -2$

Given $\frac{d^2y}{dx^2} < 0$, $(4,\ 21)$ is a maximum point.

Question 4

$y = x^3 - 9x$

$\frac{dy}{dx} = 3x^2 - 9 = 0$

$3(x^2 - 3) = 0$

$x^2 - 3 = 0$

$x = \pm\sqrt{3}$

When $x = \sqrt{3}$,

$y = (\sqrt{3})^3 - 9(\sqrt{3})$

$= -6(\sqrt{3})$

When $x = -\sqrt{3}$,

$y = 6\sqrt{3}$

Two stationary points exist at $(\sqrt{3}, -6\sqrt{3})$ and $(-\sqrt{3},\ 6\sqrt{3})$.

$\frac{d^2y}{dx^2} = 6x$

When $x = \sqrt{3}$,

$\frac{d^2y}{dx^2} = 6\sqrt{3} > 0$

$\therefore$ $(\sqrt{3}, -6\sqrt{3})$ is a minimum point.

When $x = -\sqrt{3}$,

$\frac{d^2y}{dx^2} = -6\sqrt{3} < 0$

$\therefore$ $(-\sqrt{3},\ 6\sqrt{3})$ is a maximum point.

Question 5

$$y = x^3 - 9x^2 - 21x + 60$$

$$\frac{dy}{dx} = 3x^2 - 18x - 21 = 0$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$$x = -1,\ 7$$

When $x = -1$,

$$y = (-1)^3 - 9(-1)^2 - 21(-1) + 60$$
$$= 71$$

When $x = 7$,

$$y = 7^3 - 9(7)^2 - 21(7) + 60$$
$$= -185$$

There are two stationary points at $(-1,\ 71)$ and $(7, -185)$.

When $x = -1$,

$$\frac{d^2y}{dx^2} = 6(-1) - 18 < 0$$

$\therefore (-1,\ 71)$ is a maximum point.

When $x = 7$,

$$\frac{d^2y}{dx^2} = 6(7) - 18 > 0$$

$\therefore (7, -185)$ is a minimum point.

Question 6

$y=(x-1)^4+2$

$\frac{dy}{dx}=4(x-1)^3\times 1$

$0=4(x-1)^3$

$0=(x-1)^3$

$x=1$

When $x=1$,

$y=(1-1)^4+2$

$=2$

There is a stationary point at $(1,\ 2)$.

$\frac{d^2y}{dx^2}=4\times 3\times(x-1)^2$

$=12(x-1)^2$

at $x=1$

$\frac{d^2y}{dx^2}=0$

As $f'(1)=0$ and $f''(1)=0$, we need to use the sign test.

x	0.9	1	1.1
$\frac{dy}{dx}$	−0.004	0	0.004
	\	_	/

(1, 2) is a minimum turning point

Question 7

$$y = x + 4(x+3)^{-1}$$

$$\frac{dy}{dx} = 1 + (-1)(4)(x+3)^{-2}$$

$$1 - \frac{4}{(x+3)^2} = 0$$

$$1 = \frac{4}{(x+3)^2}$$

$$4 = (x+3)^2$$

$$\pm 2 = x + 3$$

$$x = -5, -1$$

When $x = -5$,

$$y = -5 + \frac{4}{(-5+3)}$$

$$= -7$$

When $x = -1$,

$$y = -1 + \frac{4}{(-1+3)}$$

$$= 1$$

Stationary points exist at $(-5, -7)$ and $(-1, 1)$.

$$\frac{d^2y}{dx^2} = \frac{8}{(x+3)^3}$$

When $x = -5$,

$$\frac{d^2y}{dx^2} = \frac{8}{(-5+3)^3} < 0$$

$\therefore (-5, -7)$ is a maximum point.

When $x = -1$,

$$\frac{d^2y}{dx^2} = \frac{8}{(-1+3)^3} > 0$$

$\therefore (-1, 1)$ is a minimum point.

Question 8

$$y = x + \frac{5}{x}$$

$$\frac{dy}{dx} = 1 - \frac{5}{x^2} = 0$$

$$1 = \frac{5}{x^2}$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

When $x = -\sqrt{5}$,

$$y = -\sqrt{5} + \frac{5}{-\sqrt{5}}$$

$$= -2\sqrt{5}$$

When $x = \sqrt{5}$,

$$y = \sqrt{5} + \frac{5}{\sqrt{5}}$$

$$= 2\sqrt{5}$$

Stationary points exist at $(-\sqrt{5}, -2\sqrt{5})$ and $(\sqrt{5}, 2\sqrt{5})$.

$$\frac{d^2y}{dx^2} = \frac{10}{x^3}$$

When $x = -\sqrt{5}$,

$$\frac{d^2y}{dx^2} = \frac{10}{(-\sqrt{5})^3} < 0$$

$\therefore (-\sqrt{5}, -2\sqrt{5})$ is a maximum point.

When $x = \sqrt{5}$,

$$\frac{d^2y}{dx^2} = \frac{10}{(\sqrt{5})^3} > 0$$

$\therefore (\sqrt{5}, 2\sqrt{5})$ is a minimum point.

Question 9

$y=(2x-1)^5+1$

$\frac{dy}{dx}=5(2x-1)^4\times 2$

$0=10(2x-1)^4$

$0=(2x-1)^4$

$0=2x-1$

$x=\frac{1}{2}$

When $x=\frac{1}{2}$,

$y=(2\left(\frac{1}{2}\right)-1)^5+1$

$=1$

Stationary point exists at $\left(\frac{1}{2},1\right)$.

$\frac{d^2y}{dx^2}=10\times 4\times(2x-1)^3\times 2$

$=80(2x-1)^3$

When $x=\frac{1}{2}$,

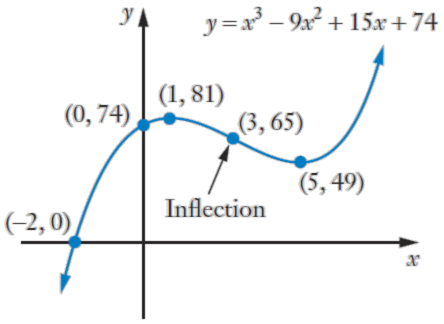
$\frac{d^2y}{dx^2}=0$

As $f'(0.5)=0$ and $f''(0.5)=0$, we need to use the sign test.

x	0.4	0.5	0.6
$\frac{dy}{dx}$	0.016	0	0.016
	/	—	/

(0.5, 1) is a point of horizontal inflection

Question 10



Question 11

a When $x = 0,\ y = 30$

$\therefore$ y-intercept at (0, 30).

b For $x \to \pm\infty,$ the x^3 will dominate.

As $x \to \infty,\ y \to \infty$ (faster than x does).

As $x \to -\infty,\ y \to -\infty$ (faster than x does).

c

$$\frac{dy}{dx} = 3x^2 - 12x - 15 = 0$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = -1,\ 5$$

When $x = -1,\ y = 38$

$x = 5,\ y = -70$

$\therefore$ Turning points at $(-1,\ 38)$ and $(5, -70)$.

$$\frac{d^2y}{dx^2} = 6x - 12$$

When $x = -1,$

$$\frac{d^2y}{dx^2} = 6(-1) - 12 < 0$$

$\therefore$ $(-1,\ 38)$ is a maximum point.

When $x = 5,$

$$\frac{d^2y}{dx^2} = 6(5) - 12 > 0$$

$\therefore$ $(5, -70)$ is a minimum point.

d $\frac{d^2y}{dx^2} = 6x - 12 = 0$

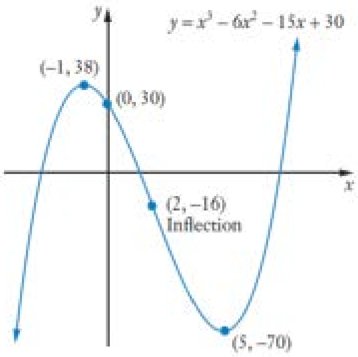
$x = 2$

When $x = 2,\ y = -16$

$\therefore (2, -16)$

As $f'(2) \neq 0,\ f''(2) = 0$

$\therefore (2, -16)$ is a point of inflection.



Question 12

$y = x^4 - 4x^3 + 1$

a When $x = 0,\ y = 1$

$\therefore$ y-intercept at (0, 1).

b For $x \to \pm\infty$, the x^4 will dominate.

As $x \to \infty,\ y \to \infty$ (faster than x does).

As $x \to -\infty,\ y \to \infty$ (faster than $x \to -\infty$).

c $\frac{dy}{dx} = 4x^3 - 12x^2 = 0$

$4x^2(x - 3) = 0$

$x = 0, 3$

When $x = 0,\ y = 1$

$x = 3,\ y = -26$

$\therefore$ Stationary points at (0, 1) and $(3, -26)$.

$$\frac{d^2y}{dx^2} = 12x^2 - 24x$$

When $x = 0$

$$\frac{d^2y}{dx^2} = 12(0)^2 - 24(0) = 0$$

As $f'(0) = 0$ and $f''(0) = 0$ we need to use the sign test.

x	-0.1	0	0.1
$\frac{dy}{dx}$	2.52	0	0.016
	/	—	/

(0, 1) is a point of horizontal inflection

When $x = 3$

$$\frac{d^2y}{dx^2} = 12(3)^2 - 24(3) > 0$$

(3, –26) is a minimum turning point

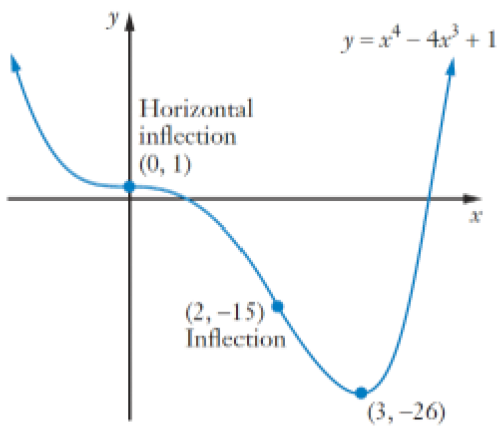
d $\frac{d^2y}{dx^2} = 12x^2 - 24x = 0$

$$12x(x-2) = 0$$

$$x = 0,\ 2$$

When $x = 2,\ y = -15$

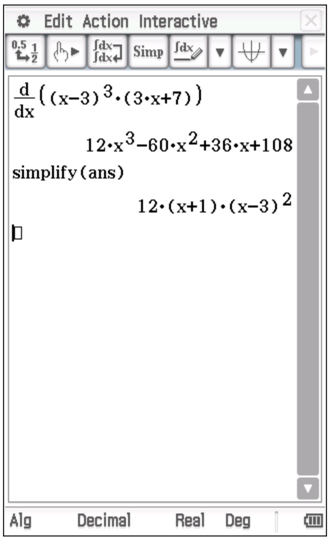
$\therefore$ (0, 1) and (2, –15) have $\frac{d^2y}{dx^2} = 0$.



Question 13

$y=(x-3)^3(3x+7)$

$$\frac{dy}{dx}=(x-3)^3\times 3+(3x+7)\times 3(x-3)^2$$
$$=3(x-3)^3\left[(x-3)+(3x+7)\right]$$
$$=3(x-3)^2(4x+4)$$
$$=3(x-3)^2\times 4(x+1)$$
$$=12(x-3)^2(x+1)$$



Stationary points: $\frac{dy}{dx}=0$

$12(x-3)^2(x+1)=0$

$x=-1,\ 3$

When $x=-1,\ y=-256$

$x=3,\ y=0$

$$\frac{d^2y}{dx^2}=12\left[(x-3)^2\times 1+(x+1)\times 2(x-3)\right]$$
$$=12(x-3)\left[(x-3)+2(x+1)\right]$$
$$=12(x-3)(3x-1)$$

When $x=-1$,

$$\frac{d^2y}{dx^2}=12(-4)(-4)>0$$

$\therefore (-1,-256)$ is a minimum point.

When $x=3$,

$$\frac{d^2y}{dx^2}=12(0)(8)=0$$

$f\,'(3)$ and $f\,''(3)=0$

$\therefore$ Need to use sign test.

x	2.9	3	3.1
$\frac{dy}{dx}$	0.468	0	0.492
	/	—	/

(3, 0) is a point of horizontal inflection.

Question 14

As $x \to +\infty$, $f'(x) \to +\infty$.
As $x \to -\infty$, $f'(x) \to -\infty$.
As $\left(\frac{4}{3}, 2\frac{5}{27}\right)$ is a local maximum, we need to investigate the behaviour of the curve when $x > 4$.

$f(5) = 1\frac{5}{8}$

a Maximum value is $2\frac{5}{27}$.

b $f(6) = 4$

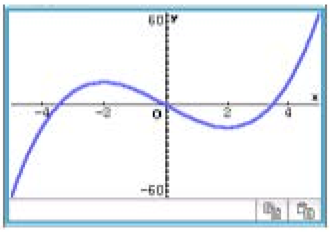
$\therefore$ Maximum value $0 \le x \le 6$ is 4.

Question 15

$$f'(x) = 3x^2 - 12$$
$$f''(x) = 6x = 0$$
$$x = 0$$

a $f(0) = 0^3 - 12(0) = 0 \to (0, 0)$

b (0, 0) is a point of inflection but not a horizontal inflection.



Question 16

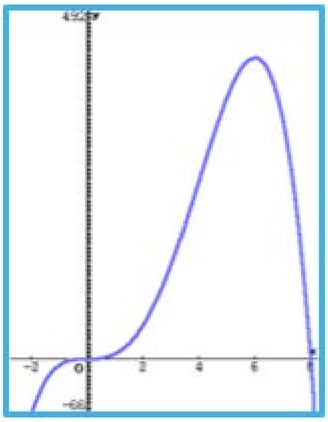
$$f'(x) = 24x^2 - 4x^3$$
$$f''(x) = 48x - 12x^2 = 0$$
$$12x(4 - x) = 0$$
$$x = 0,\ 4$$

a $f(0) = 0 \quad \rightarrow (0,\ 0)$

$f(4) = 256 \rightarrow (4,\ 256)$

b (0, 0) is a point of horizontal inflection.

(4, 256) is a point of inflection (not horizontal).

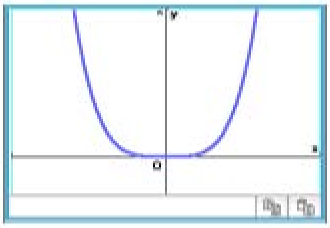


Question 17

$$f'(x) = 4x^3$$
$$f''(x) = 12x^2 = 0$$
$$x^2 = 0$$
$$x = 0$$

a $f(0) = 0^4 = 0 \rightarrow (0,0)$

b (0,0) is a minimum point



Question 18

$$f'(x) = 3x^2 - 3$$
$$f''(x) = 6x$$
$$6a = 0$$
$$a = 0$$
$$f'(0) = -2 \rightarrow (0, -2)$$

x	-0.1	0	0.1
$\frac{dy}{dx}$	-2.97	-3	-2.97
	\	—	\

$(0, -2)$ is a point of inflection but not horizontal inflection.

Exercise 2B

Question 1

$$P = 2a^3 + 3a - 7$$
$$\frac{dP}{da} = 6a^2 + 3$$

Question 2

$$Y = p - 5p^2 + 2p^3$$
$$\frac{dY}{dp} = 1 - 10p + 6p^2$$

Question 3

$$Q = (2t-1)^3$$
$$\frac{dQ}{dt} = 3(2t-1)^2 \times 2$$
$$= 6(2t-1)^2$$

Question 4

$$A = \frac{3x-2}{2x+5}$$
$$\frac{dA}{dx} = \frac{(2x+5)\times 3-(3x-2)\times 2}{(2x+5)^2}$$
$$= \frac{6x+15-6x+4}{(2x+5)^2}$$
$$= \frac{19}{(2x+5)^2}$$

Question 5

$$P = (2q-5)(3q^2+1)$$
$$\frac{dP}{dq} = (2q-5)\times 6q + 2(3q^2+1)$$
$$= 12q^2 - 30q + 6q^2 + 2$$
$$= 18q^2 - 30q + 2$$

Question 6

a

$$V = (1 + 0.5t)^3$$

$$\frac{dV}{dt} = 3(1 + 0.5t)^2 \times 0.5$$

When $t = 2$,

$$\frac{dV}{dt} = 3(1 + 1)^2 \times 0.5$$

$$= 6 \text{ cm}^3 / \text{sec}$$

b When $t = 6$,

$$\frac{dV}{dt} = 3(1 + 3)^2 \times 0.5$$

$$= 24 \text{ cm}^3 / \text{sec}$$

c When $t = 10$,

$$\frac{dV}{dt} = 3(1 + 5)^2 \times 0.5$$

$$= 54 \text{ cm}^3 / \text{sec}$$

Question 7

a

$$N = 500 - 5t^2 + 10t^3$$

$$\frac{dN}{dt} = 30t^2 - 10t$$

b **i** When $t = 1$,

$$\frac{dN}{dt} = 30(1)^2 - 10(1)$$

$$= 20 \text{ insects / day}$$

ii When $t = 5$,

$$\frac{dN}{dt} = 30(5)^2 - 10(5)$$

$$= 700 \text{ insects / day}$$

iii When $t = 10$,

$$\frac{dN}{dt} = 30(10)^2 - 10(10)$$

$$= 2900 \text{ insects / day}$$

Question 8

a When $t=1$,

$$h=5(1+2)$$
$$=15\,\text{m}$$

$$\frac{dh}{dt}=5+20t$$

When $t=1$,

$$\frac{dh}{dt}=5+20(1)$$
$$=25\ \text{m}/\text{s}$$

b When $t=5$,

$$h=25(1+10)$$
$$=275\ \text{m}$$

When $t=5$,

$$\frac{dh}{dt}=5+20(5)$$
$$=105\ \text{m}/\text{s}$$

c When $t=20$,

$$h=100(1+40)$$
$$=4100\ \text{m}$$

When $t=20$,

$$\frac{dh}{dt}=5+20(20)$$
$$=405\ \text{m}/\text{s}$$

Question 9

a $N = 5(2t+1)^3$

When $t = 0$,

$$\begin{aligned} N &= 5(2(0)+1)^3 \\ &= 5 \end{aligned}$$

b When $t = 5$,

$$\begin{aligned} N &= 5(2(5)+1)^3 \\ &= 5(11)^3 \\ &= 6655 \end{aligned}$$

c $$\begin{aligned} \frac{dN}{dt} &= 5 \times 3(2t+1)^2 \times 2 \\ &= 30(2t+1)^2 \text{ bacteria / hour} \end{aligned}$$

d **i** When $t = 2$,

$$\begin{aligned} \frac{dN}{dt} &= 30(5)^2 \\ &= 750 \text{ bacteria / hour} \end{aligned}$$

ii When $t = 5$,

$$\begin{aligned} \frac{dN}{dt} &= 30(11)^2 \\ &= 3630 \text{ bacteria / hour} \end{aligned}$$

iii When $t = 10$,

$$\begin{aligned} \frac{dN}{dt} &= 30(21)^2 \\ &= 13\ 230 \text{ bacteria / hour} \end{aligned}$$

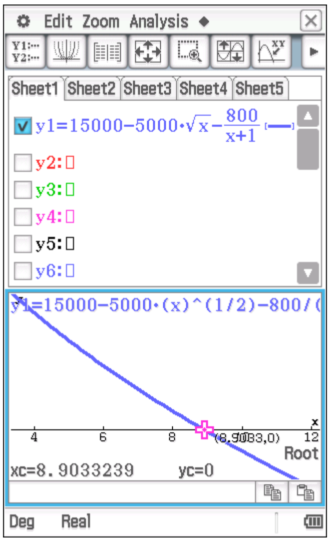
Question 10

a $R = 15000 - 5000\sqrt{w} - \dfrac{800}{w+1}$

Graph R and find the x-intercept.

x-intercept $= 8.9$

$\therefore$ After 9 weeks.



b $\dfrac{dR}{dw} = -5000 \times \dfrac{1}{2} \times w^{-\frac{1}{2}} + \dfrac{800}{(w+1)^2}$

$= \dfrac{-2500}{\sqrt{w}} + \dfrac{800}{(w+1)^2}$

At $w = 1$,

$\dfrac{dR}{dw} = \dfrac{-2500}{\sqrt{1}} + \dfrac{800}{(1+1)^2}$

$= -\$2300\,/\,\text{week}$

At $w = 3$,

$\dfrac{dR}{dw} = \dfrac{-2500}{\sqrt{3}} + \dfrac{800}{(4)^2}$

$= -1393$

$= -\$1400\,/\,\text{week}$

At $w = 8$,

$\dfrac{dR}{dw} = \dfrac{-2500}{\sqrt{8}} + \dfrac{800}{(9)^2}$

$= -874$

$= -\$900\,/\,\text{week}$

Exercise 2C

Question 1

a $+5\text{ m/s} \to +10\text{ m/s}$

$\therefore$ positive acceleration (positive direction, increasing).

b $+5\text{ m/s} \to +10\text{ m/s}$

$\therefore$ positive acceleration (positive direction, increasing).

c $+10\text{ m/s} \to +5\text{ m/s}$

$\therefore$ negative acceleration (positive direction, slowing).

d $-5\text{ m/s} \to -10\text{ m/s}$

$\therefore$ negative acceleration (negative direction, increasing).

e $-10\text{ m/s} \to -5\text{ m/s}$

$\therefore$ positive acceleration (negative direction, slowing).

f $-10\text{ m/s} \to +5\text{ m/s}$

$\therefore$ positive acceleration (changing direction from negative to positive)

Question 2

a $x = 5t^2 + 6t$

$$v = \frac{dx}{dt} = 10t + 6$$

When $t = 2$,

$$\frac{dx}{dt} = 10(2) + 6$$
$$= 26\text{ m/s}$$

b $a = \frac{dv}{dt} = 10\text{ m/s}^2$

Question 3

a $x = \frac{1}{10}(2t+1)^3$

$$v = \frac{dx}{dt} = \frac{1}{10} \times 3(2t+1)^2 \times 2$$

$$= 0.6(2t+1)^2$$

When $t = 2$,

$$v = 0.6(5)^2$$

$$= 15 \text{ m/s}$$

b $a = \frac{dv}{dt} = 1.2(2t+1) \times 2$

$$= 2.4(2t+1)$$

When $t = 2$,

$$\frac{dv}{dt} = 2.4(5)$$

$$= 12 \text{ m/s}^2$$

Question 4

$$v = \frac{2t+3}{t+1}$$

$$\frac{dv}{dt} = \frac{2(t+1) - (2t+3)}{(t+1)^2}$$

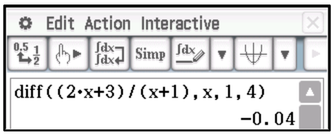
$$= \frac{2t+2-2t-3}{(t+1)^2}$$

$$= \frac{-1}{(t+1)^2}$$

When $t = 4$,

$$\frac{dv}{dt} = \frac{-1}{(4+1)^2}$$

$$= -0.04 \text{ m/s}^2$$



Question 5

$v = (2t - 1)^5$

$a = \frac{dv}{dt} = 5(2t - 1)^4 \times 2$

$= 10(2t - 1)^4$

When $t = 1.5$,

$\frac{dv}{dt} = 10(2(1.5) - 1)^4$

$= 160 \text{ m/s}^2$

Question 6

a $x = 2t^3 + 4$

$v = \frac{dx}{dt} = 6t^2$

$a = \frac{dv}{dt} = 12t$

When $t = 2$,

$a = 24 \text{ m/s}^2$

b $x = 7t$

$v = \frac{dx}{dt} = 7$

$a = \frac{dv}{dt} = 0$

When $t = 3$,

$a = 0 \text{ m/s}^2$

c $x = 27(2t + 1)^{-1}$

$v = \frac{dx}{dt} = -27(2t + 1)^{-2} \times 2$

$= -54(2t + 1)^{-2}$

$a = \frac{dv}{dt} = 108(2t + 1)^{-3} \times 2$

$= \frac{216}{(2t + 1)^3}$

When $t = 1$,

$a = \frac{216}{3^3}$

$= 8 \text{ m/s}^2$

d $x=(2t+1)^{\frac{1}{2}}$

$$v=\frac{dx}{dt}=\frac{1}{2}(2t+1)^{-\frac{1}{2}}\times 2$$

$$=(2t+1)^{-\frac{1}{2}}$$

$$a=\frac{dv}{dt}=-\frac{1}{2}(2t+1)^{-\frac{3}{2}}\times 2$$

$$=-\frac{1}{\sqrt{(2t+1)^3}}$$

When $t=4$,

$$a=-\frac{1}{\sqrt{9^3}}$$

$$=-\frac{1}{27}\ \text{m/s}^2$$

e $x=(9-2t)^4$

$$v=\frac{dx}{dt}=4(9-2t)^3\times(-2)$$

$$=-8(9-2t)^3$$

$$a=\frac{dv}{dt}=-8\times 3(9-2t)^2\times(-2)$$

$$=48(9-2t)^2$$

When $t=4$,

$$a=48(9-8)^2$$

$$=48\ \text{m/s}^2$$

f $x = 2t(1+5t)^3$

$$v = \frac{dx}{dt} = 2t \times 3(1+5t)^2 \times 5 + (1+5t)^3 \times 2$$
$$= 2(1+5t)^2[15t + (1+5t)]$$
$$= 2(1+5t)^2(20t+1)$$

$$a = \frac{dv}{dt} = 2(1+5t)^2 \times 20 + (20t+1) \times 2 \times 2(1+5t) \times 5$$
$$= 40(1+5t)^2 + 20(20t+1)(1+5t)$$
$$= 20(1+5t)[(2(1+5t) + (20t+1)]$$
$$= 20(1+5t)(3+30t)$$

When $t = 0.4$

$$a = 20 \times 3 \times 15$$
$$= 900 \text{ m/s}^2$$

Question 7

a $x = t^2 - 11t + 3$

$v = 2t - 11$

At $t = 0$,

$v = -11 \text{ m / s}$

b $a = 2 \text{ m / s}^2$

c $2t - 11 = 5$

$2t = 16$

$t = 8$

d $2t - 11 = -5$

$2t = 6$

$t = 3$

$\therefore$ Speed of 5 at $t = 3, 8$.

Question 8

$x = 27t + 3t^2 - \frac{t^3}{3} - 90$

$v = 27 + 6t - \frac{3t^2}{3}$

$\quad = 27 + 96t - t^2$

$a = 6 - 2t$

$6 - 2t = 0$

$\quad t = 3$

When $t = 3$.

$x = 81 + 27 - 9 - 90$

$= 9 \text{ m}$

Exercise 2D

Question 1

$P = 25x^2 + 5000x - x^3$

$\frac{dP}{dx} = 50x + 5000 - 3x^2$

Max profit when $\frac{dP}{dx} = 0$.

By CP,

$50x + 5000 - 3x^2 = 0$

$x = -\frac{100}{3}, 50$

Disregarding $x < 0, x = 50$

$\frac{d^2P}{dx^2} = 50 + 6x$

at $x = 50$,

$50 - 6(50) = -250$

$\therefore x = 50$ is a maximum point.

$\text{Max profit} = 25(50)^2 + 5000(50) - 50^3$
$= \$187\ 500.$

Question 2

$$P = 10\ 000x - x^3 + 275x^2 - 10^6$$

$$\frac{dP}{dx} = 10\ 000 - 3x^2 + 550x = 0$$

$$x = 200, -\frac{50}{3}$$

Disregarding $x < 0, x = 200$

$$\frac{d^2P}{dx^2} = -6x + 550$$

When $x = 200$,

$$\frac{d^2P}{dx^2} = -6(200) + 550$$

$$= -650$$

$\Rightarrow$ Maximum point at $x = 200$.

Maximum profit of $4 000 000 occurs when 200 items are made.

Question 3

$$P = -\frac{x^3}{3} + 20x^2 + 2100x - 25000$$

$$\frac{dP}{dx} = -x^2 + 40x + 2100 = 0$$

$$x = -30,\ 70$$

Disregarding $x < 0, x = 70$

When $x = 70$,

$$\frac{d^2P}{dx^2} = -2(70) + 40$$

$$= -100$$

The point $x = 70$ is a maximum point.

$\therefore$ When $x = 70,\ P = \$105\ 666.67.$

$= \$106\ 000$ (nearest $1000)

Question 4

Average weight, $w = (600 - 15N)$

Total weight, $T = (600 - 15N)N$

$$= 600N - 15N^2$$

$$\frac{dT}{dN} = 600 - 30N = 0$$

$$N = 20$$

$$\frac{d^2T}{dN^2} = -30$$

$\therefore \frac{d^2T}{dN^2} < 0$, $N = 20$ is a maximum.

Question 5

a Volume $= (25-2x)(40-2x) \times x$

$$\frac{dV}{dx} = 12x^2 - 260x + 1000 = 0$$

$$x = 5,\ 16\frac{2}{3}$$

It is not possible to form a box when $x = 16\frac{2}{3}$ $\therefore x = 5$.

$$\frac{d^2V}{dx^2} = 24x - 260$$

When $x = 5$,

$$\frac{d^2V}{dx^2} = 24(5) - 260$$
$$= -140$$

$x = 5$ is a maximum point

Max volume occurs when $x = 5$ cm.

Max volume $= (25-10)(40-10) \times 5$

$= 2250\ \text{cm}^3$

b $V = (33-2x)(40-2x) \times x$

$$\frac{dV}{dx} = 12x^2 - 292c + 1320 = 0$$

$$x = 6,\ 18\frac{2}{3}$$

It is not possible to form a box when $x = 18\frac{2}{3}$ $\therefore x = 6$.

$$\frac{d^2V}{dx^2} = 24x - 292$$

When $x = 6$,

$$\frac{d^2V}{dx^2} = 24(6) - 292$$
$$= -148$$

$\therefore$ as $\frac{d^2V}{dx^2} < 0$, when $x = 6$, maximum volume $= 3528\ \text{cm}^3$.

Question 6

$C = 0.025x^2 + 2x + 1000,\ x > 0$

Average cost = cost ÷ number of items

$AC = 0.025x + 2 + \frac{1000}{x}$

$\frac{dAC}{dx} = 0.025 - \frac{1000}{x^2} \quad = 0$

$x^2 = 40000$

$x = \pm 200$

$x > 0 \therefore x = 200$

$\frac{d^2AC}{dx^2} = \frac{2000}{x^3}$

When $x = 200$

$\frac{d^2AC}{dx^2} = \frac{2000}{200^3} > 0$

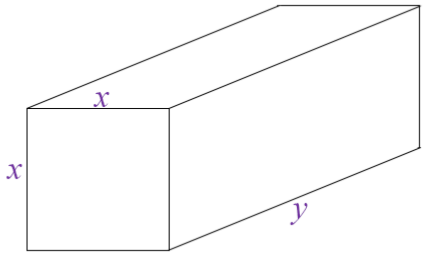
As $\frac{d^2C}{dx^2} > 0,\ x = 200$ is a minimum point.

Minimum average cost

$= 0.025(200) + 2 + \frac{1000}{(200)}$

$= \$12$

Question 7



$x \times x \times y = 1000 \text{ cm}^3$

$x^2 y = 1000$

$y = \dfrac{1000}{x^2}$

$SA = 2x^2 + 4xy$

$= 2x^2 + 4x \times \dfrac{1000}{x^2}$

$= 2x^2 + \dfrac{4000}{x}$

$\dfrac{dSA}{dx} = 4x - \dfrac{4000}{x^2} = 0$

By ClassPad $x = 10$.

$\dfrac{d^2SA}{dx^2} = 4 + \dfrac{8000}{x^3}$

When $x = 10, \dfrac{d^2SA}{dx^2} = 4 + \dfrac{8000}{10^3}$

$= 12$

As $\dfrac{d^2SA}{dx^2} > 0$ when $x = 10$, $x = 10$ is a minimum point.

$y = \dfrac{1000}{x^2} = \dfrac{1000}{100} = 10$

$\therefore$ All dimensions are 10 cm.

Question 8

Profit:

A:\$5600

B:\$200

Items produced:

$x\text{A} + (400 - x^2)\text{B}$

Profit:

$$\begin{aligned}\text{P} &= 5600x + 200(400 - x^2)\\ &= 5600x + 80000 - 200x^2\end{aligned}$$

$$\frac{d\text{P}}{dx} = 5600 - 400x = 0$$

$$x = 14$$

$$\frac{d^2\text{P}}{dx^2} = -400$$

When $x = 14, \dfrac{d^2\text{P}}{dx^2} < 0$

$\therefore$ P is a maximum.

$$\begin{aligned}\text{P} &= 5600(14) + 200(400 - 14^2)\\ &= \$119200\end{aligned}$$

$\therefore$ Max of \$119 200 when there are 14 As and 204 Bs.

Question 9

$2\pi r + y = 120$

$y = 120 - 2\pi r$

$V = \pi r^2 y$

$= \pi r^2(120 - 2\pi r)$

$= 120\pi r^2 - 2\pi^2 r^3$

$\frac{dV}{dr} = 240\pi r - 6\pi^2 r^2 = 0$

By ClassPad $r = 0, \frac{40}{\pi}$

$\Rightarrow r = \frac{40}{\pi}$

$\frac{d^2V}{dr^2} = 240\pi - 12\pi^2 r$

When $r = \frac{40}{\pi}$,

$\frac{d^2V}{dr^2} = 240\pi - 12\pi^2 \times \frac{40}{\pi}$

$= 240\pi - 480\pi$

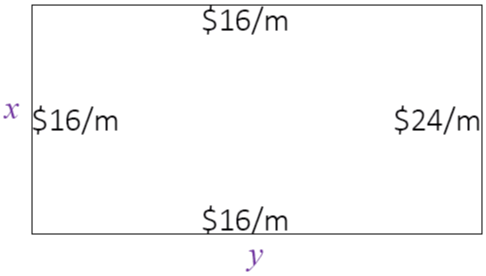
$= -240\pi$

When $r = \frac{40}{\pi}$, it is a maximum point.

$y = 120 - 2\pi(\frac{40}{\pi})$

$= 40$

Question 10



$xy = 8000$

$y = \frac{8000}{x}$

$$\text{Cost} = 16x + 16y + 16y + 24x$$
$$= 40x + 32y$$
$$= 40x + 32\left(\frac{8000}{x}\right)$$
$$= 40x + 256000x^{-1}$$

$$C'(x) = 40 - \frac{256000}{x^2} = 0$$
$$40 = \frac{256000}{x^2}$$
$$40x^2 = 256000$$
$$x^2 = 6400$$
$$x = \pm 80$$
$$\Rightarrow x = 80$$
$$\therefore y = \frac{8000}{80}$$
$$= 100$$

$$\therefore \text{Cost} = 40(80) + 32(100)$$
$$= \$6400$$

Check for minimum,

$$\frac{d^2C}{dx^2} = \frac{256\ 000}{x^3} > 0 \quad \checkmark$$

Question 11

$P = \dfrac{15x}{64 + x^2}$

$\dfrac{dP}{dx} = \dfrac{-(15x^2 - 960)}{(x^2 + 64)^2} = 0$

By ClassPad $x = -8,\ 8$

$x > 0 \Rightarrow x = 8$

$\dfrac{d^2P}{dx^2} = \dfrac{30x^2 - 5760x}{(x^2 + 64)^3}$

When $x = 8$,

$\dfrac{d^2P}{dx^2} = \dfrac{-15}{1024} < 0$

$\therefore$ Maximum value occurs when $x = 8$.

$P = \dfrac{15(8)}{64 + 8^2}$

$= \dfrac{120}{128}$ or 0.9375.

Question 12

$P = 18t(t^2 + 5t + 100)^{-1}$

$\dfrac{dP}{dt} = \dfrac{18(100 - t^2)}{(t^2 + 5t + 100)^2} = 0$

$18(100 - t^2) = 0$

$t = \pm 10$

As $t > 0, t = 10$

$\dfrac{d^2P}{dt^2} = \dfrac{-36(300t - t^3 + 500)}{(t^2 + 5t + 100)^3}$

When $t = 10$,

$\dfrac{d^2P}{dt^2} = -0.00576$

$\therefore t = 10$ is a maximum.

$P = 18(10)(100 + 50 + 100)^{-1}$

$= 0.72$

Question 13

$C = (x+10)^3$

Average Cost $= \dfrac{(x+10)^3}{x}$

$$\frac{d\ \text{AC}}{dx} = \frac{x \times 3(x+10)^2 - (x+10)^3 \times 1}{x^2}$$

$$= \frac{(x+10)^2(3x-(x+10))}{x^2}$$

$$= \frac{(x+10)^2(2x-10)}{x^2}$$

$$0 = (x+10)^2(2x-10)$$

$$x = 5, -10$$

$$x = 5\ (x > 0)$$

When $x = 5$

Average cost $= \dfrac{15^3}{5}$

$= \$675$

$$\frac{d^2\text{AC}}{dx^2} = \frac{x^2(6x^2+60x)-(2x^3+30x^2-1000)}{x^4}$$

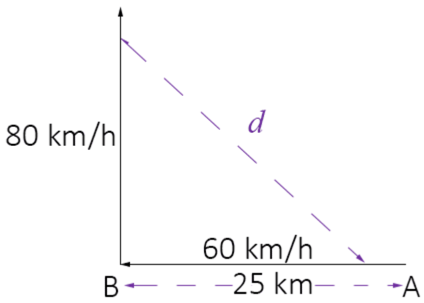
$$= \frac{24x^4+60x^3-2x^3-30x^2+1000}{x^4}$$

$$= \frac{24x^4+58x^3-30x^2+1000}{x^4}$$

At $x = 5$, $\dfrac{d^2\text{AC}}{dx^2} = 36 > 0$

$\therefore$ Cost is a minimum.

Question 14



After t hours, A is $(25-60t)$ km away from B's initial posi

B is $80t$ km North of its initial position.

They are $\sqrt{(25-60t)+80t^2}$ km apart.

$$d=(625-3000t+10\ 000t^2)^{\frac{1}{2}}$$

$$\frac{dd}{dt}=\frac{1}{2}(625-3000t+10\ 000t^2)^{-\frac{1}{2}}\times(-3000+20\ 000t)$$

$$\frac{dd}{dt}=\frac{-(1500-100\ 00t)}{(625-3000t+10\ 000t^2)^{\frac{1}{2}}}=0$$

$$-1500+10\ 000t=0$$

$$100t=15$$

$$t=\frac{15}{100}$$

$$t=0.15 \text{ hour}$$

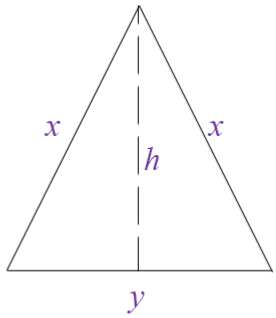
$$=9 \text{ mins.}$$

Separation distance: $\sqrt{625-3000(0.15)+10000(0.15)^2}$

$= 20$ km.

Question 15

$p = 2x + y \Rightarrow y = p - 2x$ $*p$ is a fixed value ie a constant



$h^2 = x^2 - \left(\frac{y}{2}\right)^2$

$h = \sqrt{x^2 - \left(\frac{y}{2}\right)^2}$

$A = \frac{1}{2} \times y \times h$

$= \frac{1}{2} \times (p - 2x) \times \sqrt{x^2 - \left(\frac{p-2x}{2}\right)^2}$

By Classpad

$\frac{dA}{dx} = \frac{p^2 - 3px}{\sqrt{p(4x - p)}}$

Solving $\frac{dA}{dx} = 0$

$p^2 - 3px = 0$

$x = \frac{p}{3} \Rightarrow p = 3x$

The base must have the same length as congruent sides.

The triangle must be equilateral to maximise area.

Exercise 2E

Question 1

$f(x) = x^2 + 4x$

$f'(x) = 2x + 4$

$\delta x = 0.02$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$$\begin{aligned}\delta y &\approx \frac{dy}{dx} \times \delta x \\ &\approx (2 \times 4 + 4) \times 0.02 \\ &\approx 0.24\end{aligned}$$

An approximate increase of 0.24.

$$\begin{aligned}f(4.02) - f(4) &= 32.2404 - 32 \\ &= 0.2404\end{aligned}$$

Question 2

$f(x) = 2x^2 - 5x$

$\frac{dy}{dx} = 4x - 5$

$\delta x = 0.01$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$$\begin{aligned}\delta y &\approx \frac{dy}{dx} \times \delta x \\ &\approx (4 \times 3 - 5) \times 0.01 \\ &\approx 0.07\end{aligned}$$

An approximate increase of 0.07

$$\begin{aligned}f(3.01) - f(3) &= 3.0702 - 3 \\ &= 0.0702\end{aligned}$$

Question 3

$$y = x^3 + 4$$

$$\frac{dy}{dx} = 3x^2$$

$$\delta x = 0.05$$

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$

$$\begin{aligned}\delta y &\approx \frac{dy}{dx} \times \delta x \\ &\approx 3 \times 1^2 \times 0.05 \\ &\approx 0.15\end{aligned}$$

An approximate increase of 0.15.

Question 4

$$y = 2x^3 - 4x$$

$$\frac{dy}{dx} = 6x^2 - 4$$

$$\delta x = 0.01$$

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$

$$\begin{aligned}\delta y &\approx \frac{dy}{dx} \times \delta x \\ &\approx (6 \times (5)^2 - 4) \times 0.01 \\ &\approx 1.46\end{aligned}$$

An approximate increase of 1.46.

Question 5

$$y = t^3 + 3t^2 - 6t + 4$$
$$\frac{dy}{dx} = 3t^2 + 6t - 6$$
$$\delta x = 0.01$$
$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$
$$\begin{aligned}\delta y &\approx \frac{dy}{dx} \times \delta x \\ &\approx (3(2)^2 + 6(2) - 6) \times 0.01 \\ &\approx 0.18\end{aligned}$$

An approximate increase of 0.18.

Question 6

$$y = (t+1)^{-1}$$
$$\frac{dy}{dx} = -1(t+1)^{-2} = \frac{-1}{(t+1)^2}$$
$$\delta x = 0.1$$
$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$
$$\begin{aligned}\delta y &\approx \frac{dy}{dx} \times \delta x \\ &\approx \frac{-1}{(4+1)^2} \times 0.1 \\ &\approx -0.004\end{aligned}$$

An approximate decrease of 0.004.

Question 7

$$y = t^{\frac{1}{2}}$$

$$\frac{dy}{dt} = \frac{1}{2} \times \frac{1}{\sqrt{t}}$$

$$\delta t = 1$$

$$\frac{\delta y}{\delta t} \approx \frac{dy}{dt}$$

$$\begin{aligned} \delta y &\approx \frac{dy}{dt} \times \delta t \\ &\approx \frac{1}{2(5)} \times 1 \\ &= 0.1 \end{aligned}$$

An approximate increase of 0.1.

Question 8

$$y = 3x^2$$

$$\frac{dy}{dx} = 6x$$

$$\frac{\delta x}{x} = 0.05$$

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

$$\begin{aligned} \frac{\delta y}{y} &\approx \frac{dy}{dx} \times \frac{\delta x}{y} \\ &\approx \frac{6x \times \delta x}{3x^2} \\ &\approx \frac{6x \times \ \delta x}{3x \times x} \\ &\approx 2 \times 0.05 \\ &\approx 10\% \end{aligned}$$

An approximate increase of 10%.

Question 9

$$y = t^3$$

$$\frac{dy}{dt} = 3t^2$$

$$\frac{\delta t}{t} = 0.02$$

$$\frac{\delta y}{\delta t} \approx \frac{dy}{dt}$$

$$\delta y \approx \frac{dy}{dt} \times \delta t$$

$$\frac{\delta y}{y} \approx \frac{dy}{dx} \times \frac{\delta t}{y}$$

$$\approx 3t^2 \times \frac{\delta t}{t^3}$$

$$\approx \frac{3t^2}{t^2} \times \frac{\delta t}{t}$$

$$\approx 3 \times 0.02$$

$$\approx 6\%$$

An approximate increase of 6%.

Question 10

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\delta r = 0.1$$

$$\frac{\delta A}{\delta r} \approx \frac{dA}{dr}$$

$$\delta A \approx \frac{dA}{dr} \times \delta r$$

$$\approx 2\pi \times 10 \times 0.1$$

$$\approx 2\pi \text{ cm}^2$$

An approximate increase of 2π cm^2.

Question 11

$$A = \pi r^2$$

$$120 = \pi r^2$$

$$r = \sqrt{\frac{120}{\pi}}$$

$$= 6.180 \text{ cm (3 dp)}$$

$$\frac{dA}{dr} = 2\pi r$$

$$\delta A = 1$$

$$\frac{\delta A}{\delta r} \approx \frac{dA}{dr}$$

$$\frac{\delta r}{\delta A} \approx \frac{dr}{dA}$$

$$\delta r \approx \frac{dr}{dA} \times \delta A$$

$$\approx \frac{1}{2\pi \times 6.180} \times 1$$

$$\approx 0.026 \text{ cm}$$

An approximate increase of 0.026 cm.

Question 12

$$C = n^3 - 45n^2 + 800n + 1000$$

$$\frac{dC}{dn} = 3n^2 - 90n + 800$$

$$\delta n = 1$$

$$\frac{\delta C}{\delta n} \approx \frac{dC}{dn}$$

$$\delta C \approx \frac{dC}{dn} \times \delta n$$

$$\approx 3(20)^2 - 90(20) + 800$$

$$\approx \$200$$

An approximate increase of $200.

Question 13

$R = 25x - 0.01x^2$

$\frac{dR}{dx} = 25 - 0.02x$

$\delta x = 1$

$\frac{\delta R}{\delta x} \approx \frac{dR}{dx}$

$$\begin{aligned}\delta R &\approx \frac{dR}{dx} \times \delta x \\ &\approx (25 - 0.02 \times 200) \times 1 \\ &\approx \$21\end{aligned}$$

An approximate increase of $21.

Question 14

$SA = 4\pi r^2$

$\frac{dSA}{dr} = 8\pi r$

$\delta r = -0.01$

$\frac{\delta SA}{\delta r} \approx \frac{dSA}{dr}$

$$\begin{aligned}\delta SA &\approx \frac{dSA}{dr} \times \delta r \\ &\approx 8\pi \times 10 \times (-0.01) \\ &\approx -0.8\pi\end{aligned}$$

An approximate decrease of 0.8π cm^2.

Question 15

$$A = kW^{0.4}$$

$$\frac{dA}{dW} = k \times 0.4W^{-0.6}$$

$$\frac{\delta W}{W} = 0.02$$

$$\frac{\delta A}{\delta W} \approx \frac{dA}{dW}$$

$$\delta A \approx \frac{dA}{dW} \times \delta W$$

$$\frac{\delta A}{A} \approx \frac{dA}{dW} \times \frac{\delta W}{A}$$

$$\approx \frac{k \times 0.4W^{-0.6} \times \delta W}{kW^{0.4}}$$

$$\approx \frac{0.04 \times \delta W}{W^{0.4} \times W^{0.6}}$$

$$\approx 0.04 \times \frac{\delta W}{W}$$

$$\approx 0.4 \times 0.02$$

$$\approx 0.008$$

An approximate increase of 0.8%.

Question 16

$$SA = 4\pi r^2$$

$$\frac{dSA}{dr} = 8\pi r$$

$$\delta r = 0.1$$

$$\frac{\delta SA}{\delta r} \approx \frac{dSA}{dr}$$

$$\delta SA \approx \frac{dSA}{dr} \times \delta r$$

$$\approx 8\pi \times 2.5 \times 0.1$$

$$\approx 2\pi \text{ cm}^2$$

An approximate increase of 2π cm^2.

Question 17

$$P = 20x^2 - 4000 - \frac{x^3}{12}$$
$$\frac{dP}{dx} = 40x - \frac{x^2}{4}$$
$$\delta x = 1$$
$$\frac{\delta P}{\delta x} \approx \frac{dP}{dx}$$
$$\begin{aligned} \delta P &\approx \frac{dP}{dx} \times \delta x \\ &\approx \left(40(100) - \frac{100^2}{4}\right) \times 1 \\ &\approx \$1500 \end{aligned}$$

An approximate increase of \$1500.

Question 18

$$V = \frac{4}{3}\pi r^3$$
$$\begin{aligned} 288\pi &= \frac{4\pi r^3}{3} \\ r^3 &= \frac{3}{4} \times 288 \\ r &= 6 \end{aligned}$$
$$\frac{dV}{dr} = 4\pi r^2 \Rightarrow \frac{dr}{dV} = \frac{1}{4\pi r^2}$$
$$\delta V = 5$$
$$\frac{\delta V}{\delta r} \approx \frac{dV}{dr}$$
$$\frac{\delta r}{\delta V} \approx \frac{dr}{dV}$$
$$\begin{aligned} \delta r &\approx \frac{dr}{dV} \times \delta V \\ &\approx \frac{1}{4\pi r^2} \times \delta V \\ &\approx \frac{1}{4\pi \times 6^2} \times 5 \\ &\approx 0.011 \text{ cm} \end{aligned}$$

$\therefore$ For spheres of volume $288\pi \pm 5$ cm^3, you need radii of 6 ± 0.011 cm.

Question 19

$$V = \pi r^2 h$$
$$= \pi r^2 \times 0.05$$
$$= 0.05\pi r^2$$
$$\frac{dV}{dr} = 0.1\pi r \Rightarrow \frac{dr}{dV} = \frac{10}{\pi r}$$
$$\delta V = 1$$
$$\frac{\delta V}{\delta r} \approx \frac{dV}{dr}$$
$$\frac{\delta r}{\delta V} \approx \frac{dr}{dV}$$
$$\delta r \approx \frac{dr}{dV} \times \delta V$$
$$\approx \frac{10}{\pi \times 20} \times 1$$
$$\approx \frac{1}{2\pi} \text{ m}$$
$$\approx 0.159 \text{ m}$$
$$\approx 16 \text{ cm}$$

An approximate increase of 16 cm.

Question 20

$$V = l^3$$
$$\frac{dV}{dl} = 3l^2$$
$$\delta l = 0.4 \qquad \text{(Subtract 2 mm from each end)}$$
$$\frac{\delta V}{\delta l} \approx \frac{dV}{dl}$$
$$\delta V \approx \frac{dV}{dl} \times \delta l$$
$$\approx 3 \times 10^2 \times 0.4$$
$$\approx 120 \text{ cm}^3$$

Approximately 120 cm^3 required.

Question 21

$$T = 2\pi \times \frac{\sqrt{l}}{\sqrt{g}}$$

$$\frac{dT}{dl} = \frac{2\pi}{\sqrt{g}} \times \frac{1}{2} l^{-\frac{1}{2}}$$

$$= \frac{\pi}{\sqrt{l} \times \sqrt{g}}$$

$$= \frac{\pi}{\sqrt{lg}}$$

$$\frac{\delta l}{l} = 0.06$$

$$\frac{\delta T}{\delta l} \approx \frac{dT}{dl}$$

$$\delta T \approx \frac{dT}{dl} \times \delta l$$

$$\frac{\delta T}{T} \approx \frac{dT}{dl} \times \frac{\delta l}{T}$$

$$\approx \frac{\pi}{\sqrt{lg}} \times \delta l \div \frac{2\pi\sqrt{l}}{\sqrt{g}}$$

$$\approx \frac{\pi \times \delta l \times \sqrt{g}}{\sqrt{lg} \times 2\pi\sqrt{l}}$$

$$\approx \frac{1}{2} \times \frac{\delta l}{l}$$

$$\approx \frac{1}{2} \times 0.06$$

$$\approx 0.03$$

An approximate increase of 3%.

Miscellaneous exercise two

Question 1

a $\sqrt{8} = \sqrt{4} \times \sqrt{2}$

$= 2\sqrt{2}$

b $\sqrt{32} = \sqrt{16} \times \sqrt{2}$

$= 4\sqrt{2}$

c $\sqrt{50} = \sqrt{25} \times \sqrt{2}$

$= 5\sqrt{2}$

d $\sqrt{18} = \sqrt{9} \times \sqrt{2}$

$= 3\sqrt{2}$

e $\sqrt{98} + 3\sqrt{2} = \sqrt{49} \times \sqrt{2} + 3\sqrt{2}$

$= 7\sqrt{2} + 3\sqrt{2}$

$= 10\sqrt{2}$

f $\sqrt{200} - \sqrt{72} = \sqrt{100} \times \sqrt{2} - \sqrt{36} \times \sqrt{2}$

$= 10\sqrt{2} - 6\sqrt{2}$

$= 4\sqrt{2}$

g $\frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2}$

$= 2\sqrt{2}$

h $5\sqrt{2} - \frac{2}{\sqrt{2}} = 5\sqrt{2} - \sqrt{2}$

$= 4\sqrt{2}$

i $\frac{20}{\sqrt{2}} - \sqrt{128} = \frac{20\sqrt{2}}{\sqrt{2} \times \sqrt{2}} - \sqrt{64} \times \sqrt{2}$

$= 10\sqrt{2} - 8\sqrt{2}$

$= 2\sqrt{2}$

Question 2

$$\begin{aligned} y &= x^3x^2 \\ \frac{dy}{dx} &= x^3 \times 2x + x^2 \times 3x^2 \\ &= 2x^4 + 3x^4 \\ &= 5x^4 \end{aligned}$$

Question 3

$$\begin{aligned} y &= \frac{x^7}{x^2} \\ \frac{dy}{dx} &= \frac{x^2 \times 7x^6 - x^7 \times 2x}{x^4} \\ &= \frac{7x^8 - 2x^8}{x^4} \\ &= \frac{5x^8}{x^4} \\ &= 5x^4 \end{aligned}$$

Question 4

$20x$

Question 5

5

Question 6

2

Question 7

$6x^2 - 6x$

Question 8

$15x^2 + 14x - 6$

Question 9

0

Question 10

$$(2x-1)\times 3+(3x+2)\times 2$$
$$=6x-3+6x+4$$
$$=12x+1$$

Question 11

$$3x^2(2)+(2x-1)\times 6x$$
$$=6x^2+12x^2-6x$$
$$=18x^2-6x$$

Question 12

$$2(2x+5)\times 2$$
$$=4(2x+5)$$
$$=8x+20$$

Question 13

$$7(2x-1)^6\times 2$$
$$=14(2x-1)^6$$

Question 14

$$\frac{(2x-3)\times 5-(5x+1)\times 2}{(2x-3)^2}$$
$$=\frac{10x-15-10x-2}{(2x-3)^2}$$
$$=-\frac{17}{(2x-3)^2}$$

Question 15

$$\frac{(3x^2-1)\times 5-(5x+1)\times 6x}{(3x^2-1)^2}$$
$$=\frac{15x^2-5-30x^2-6x}{(3x^2-1)^2}$$
$$=\frac{-15x^2-6x-5}{(3x^2-1)^2}$$
$$=\frac{-(15x^2+6x+5)}{(3x^2-1)^2}$$
$$=-\frac{(15x^2+6x+5)}{(3x^2-1)^2}$$

Question 16

$$\frac{dx}{dt}=15t^2+3$$
$$\frac{d^2x}{dt^2}=30t$$
When $t=3$,
$$\frac{d^2x}{dt^2}=30\times 3$$
$$=90$$

Question 17

$$\frac{dx}{dt}=6t+\frac{2}{t^2}$$
$$\frac{d^2x}{dt^2}=6-\frac{4}{t^3}$$
When $t=2$,
$$\frac{d^2x}{dt^2}=6-\frac{4}{2^3}$$
$$=5.5$$

Question 18

$$\frac{dx}{dt} = 5(2t+3)^4 \times 2$$
$$= 10(2t+3)^4$$
$$\frac{d^2x}{dt^2} = 10 \times 4(2t+3)^3 \times 2$$
$$= 80(2t+3)^3$$

When $t = 1$,

$$\frac{d^2x}{dt^2} = 80(2(1)+3)^3$$
$$= 10\ 000$$

Question 19

$$\frac{dx}{dt} = 3t^2 + 40t - \frac{1}{2} \times 500t^{-\frac{1}{2}}$$
$$= 3t^2 + 40t - \frac{250}{t^{\frac{1}{2}}}$$
$$\frac{d^2x}{dt^2} = 6t + 40 + \frac{125}{t^{\frac{3}{2}}}$$

When $t = 25$,

$$\frac{d^2x}{dt^2} = 6(25) + 40 + \frac{125}{25^{\frac{3}{2}}}$$
$$= 191$$

Question 20

$$A = 3x^2(25-2x)^5$$

$$\frac{dA}{dx} = 3[x^2 \times 5(25-2x)^4(-2) + (25-2x)^5 \times 2x]$$

$$= 3[-10x^2(25-2x)^4 + 2x(25-2x)^5]$$

$$= 3[2x(25-2x)^4(-5x+(25-2x))]$$

$$= 6x(25-2x)^4(-7x+25)$$

$$= 6x(25-2x)^4(25-7x)$$

$$\frac{dA}{dx} = 0$$

$\therefore$ Stationary points exist at $x = 0, \dfrac{25}{7}, 12.5$

$\dfrac{d^2A}{dx^2} = -6(84x^2 - 600x + 625)(2x - 25x^2)$ By ClassPad

at $x = 0$, $\dfrac{d^2A}{dx^2} > 0$

$\therefore$ At $x = 0$ a minimum point exists.

at $x = \dfrac{25}{7}$, $\dfrac{d^2A}{dx^2} < 0$

$\therefore$ At $x = \dfrac{25}{7}$ a maximum point exists.

at $x = 12.5$, $\dfrac{d^2A}{dx^2} = 0$

$\therefore$ We need to use the sign test as $f\,'(12.5) = 0$ and $f\,''(12.5) = 0$.

x	12.4	12.5	12.6
$\dfrac{dA}{dx}$	$-ve$	0	$-ve$
	\	—	\

$\therefore$ At $x = 12.5$, a point of horizontal inflection exists.

Question 21

$y = (x-1)(x^2 - x - 5)$

$\frac{dy}{dx} = (x-1)(2x-1) + (x^2 - x - 5) \times 1$

$= 2x^2 - 3x + 1 + x^2 - x - 5$

$= 3x^2 - 4x - 4$

When $x = 1$,

$\frac{dy}{dx} = 3(-1)^2 - 4(-1) - 4$

$= 3$

$\therefore$ Tangent is of the form $y = 3x + c$.

Using $(-1, 6)$

$6 = 3(-1) + c$

$c = 9$

$\therefore y = 3x + 9$

Question 22

a $R = \frac{q^2(400-q)}{2}$

$= 200q^2 - \frac{q^3}{2}$

$\frac{dR}{dq} = 400q - \frac{3q^2}{2}$

When $q = 50$,

$\frac{dR}{dq} = 400(50) - \frac{3(50)^2}{2}$

$= 16\ 250$

b When $q = 100$,

$\frac{dR}{dq} = 400(100) - \frac{3(100)^2}{2}$

$= 25\ 000$

Question 23

a $y = \dfrac{x^2+2}{x-1}$

$$\frac{dy}{dx} = \frac{(x-1)(2x)-(x^2+2)\times 1}{(x-1)^2}$$

$$= \frac{2x^2-2x-x^2-2}{(x-1)^2}$$

$$= \frac{x^2-2x-2}{(x-1)^2}$$

b $\dfrac{dy}{dx} = 0$

By ClassPad, $x = 1-\sqrt{3}, 1+\sqrt{3}$

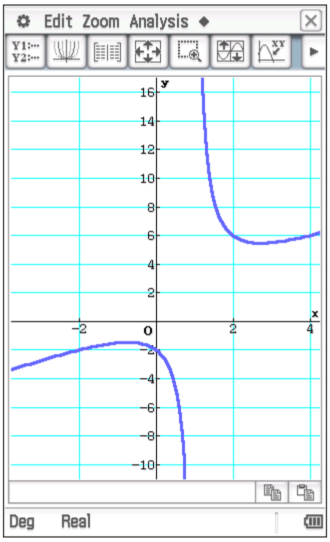
When $x = 1-\sqrt{3}$, $y = 2-2\sqrt{3}$

When $x = 1+\sqrt{3}$, $y = 2+2\sqrt{3}$

$\therefore$ Stationary points at $(1-\sqrt{3}, 2-2\sqrt{3})$ and $(\sqrt{3}+1,\ 2+2\sqrt{3})$

c $(1-\sqrt{3}, 2-2\sqrt{3})$ is a maximum point.

$(\sqrt{3}+1,\ 2+2\sqrt{3})$ is a minimum point.



Question 24

$39 = a(3)^2 + b \quad \Rightarrow \; 9a + b = 39 \quad \rightarrow$ Equation 1

$c = a(-2)^2 + b \quad \Rightarrow \quad 4a + b = c \quad \rightarrow$ Equation 2

$y - 30x + 7 = 0$

$y = 30x - 7$

$m = 30$

$\frac{dy}{dx} = 2ax$

When $x = 3$, $\frac{dy}{dx} = 30$

$30 = 2a(3)$

$6a = 30$

$a = 5$

Using Equation 1,

$9a + b = 39$

$9(5) + b = 39$

$45 + b = 39$

$b = -6$

Using Equation 2,

$4a + b = c$

$4(5) - 6 = c$

$c = 14$

Equation of curve $y = 5x^2 - 6$

$\frac{dy}{dx} = 10x$

At $B(-2, 14)$,

$\frac{dy}{dx} = 10(-2)$

$= -20$

$\therefore$ Equation of tangent $y = -20x + c$.

Using $(-2, 14)$

$14 = -20(-2) + c$

$14 = 40 + c$

$c = -26$

$\therefore$ Equation of tangent $y = -20x - 26$.

Question 25

$y = \frac{20}{x}$

$\frac{dy}{dx} = \frac{-20}{x^2}$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y \approx \frac{dy}{dx} \times \delta x$

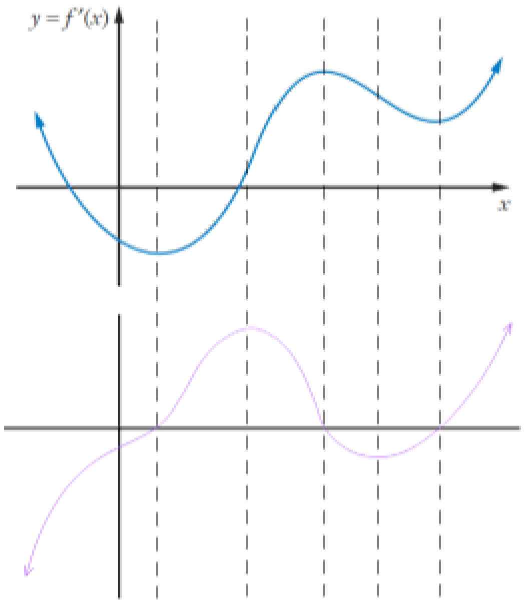
$\frac{\delta y}{y} \approx \frac{dy}{dx} \times \frac{\delta x}{y}$

$$\begin{aligned}\frac{\delta y}{y} &\approx \frac{-20}{x^2} \times \delta x \div \frac{20}{x}\\ &\approx \frac{-20}{x} \times \frac{\delta x}{x} \times \frac{x}{20}\\ &\approx -\frac{\delta x}{x}\\ &\approx -0.02\end{aligned}$$

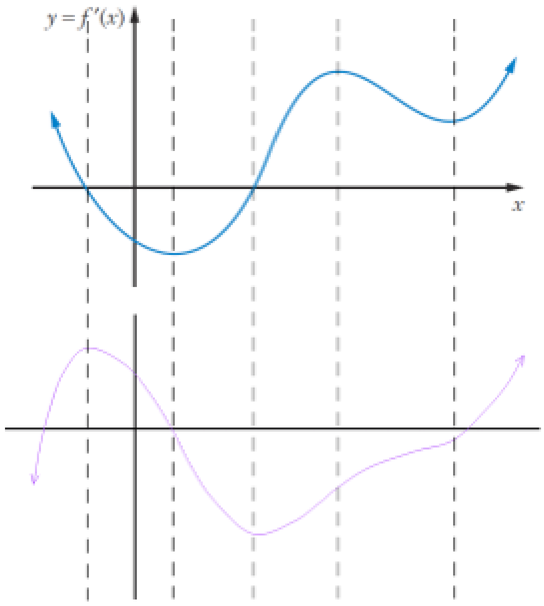
$\therefore$ Approximate decrease in y by 2%.

Question 26

a



b



Question 27

See answer in text book.